Worksheet 3b

1. Revisit Example 3.24

Read through Example 3.24 again and complete the following questions:

a. In the base case, where do the LHS (1) and RHS ($\frac{1(1+1)}{2}$) of the equation come from?

b. What can we assume in the inductive step? Why?

c. After the assumption, what is the goal of our inductive step?

Assumption	Goal

d.

$P(k+1)$		Why would we do that?
$1+2+\cdots+k+(k+1)$	$=[1+2+\cdots+k]+(k+1)$	
	$=\left[\frac{k(k+1)}{2}\right]+(k+1)$	
	$=\frac{k^2+k}{2}+\frac{2k+2}{2}$	
	$=\frac{(k+1)(k+2)}{2}$	
	$=\frac{(k+1)((k+1)+1)}{2}$	

2. Revisit Challenge 3.33

Here is a list of numbers that generated by $n^2 + n + 41$. I have checked the first 39 for you, fill in the last two rows of the table.

n	$n^2 + n + 41$	Prime or Not prime
1	43	prime
2	47	prime
3	53	prime
4	61	prime
5	71	prime
6	83	prime
7	97	prime
8	113	prime
9	131	prime
10	151	prime
11	173	prime
12	197	prime
13	223	prime
14	251	prime
15	281	prime
16	313	prime
17	347	prime
18	383	prime
19	421	prime
20	461	prime

21	503	prime
22	547	prime
23	593	prime
24	641	prime
25	691	prime
26	743	prime
27	797	prime
28	853	prime
29	911	prime
30	971	prime
31	1033	prime
32	1097	prime
33	1163	prime
34	1231	prime
35	1301	prime
36	1373	prime
37	1447	prime
38	1523	prime
39	1601	prime
40		
41		

What can you conclude? What can you learn from this?

3. Prove that for all natural number n, $\frac{1}{2*3}+\frac{1}{3*4}+\cdots+\frac{1}{(n+1)*(n+2)}=\frac{n}{2n+4}$

    a. Base case:

    b. Inductive step:

       (Start with "Assume … is true")

4. Prove that for all natural number n, $8|(5^{2n} - 1)$

5. For fun: Prove Example 3.24 without induction

6. Prove that for all natural number n, $2^1 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 2$

7. Prove that for all natural number n, $\sum_{i=1}^{n}(8i-5)=4n^2-n$